Using Numbers to Structure Space

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Abstract
We investigated the claim that relational language promotes the development of relational reasoning (Gentner, 2003). Prior research has shown the benefit of spatial relational language (e.g. top, middle, bottom) in preschoolers’ performance in spatial mapping (Loewenstein and Gentner, 2005), suggesting that spatial relational language invites a delineated relational representation. We generalized this conclusion by testing the benefit of using nonspatial relational language in a spatial analogical task. Preschool children were presented with two identical three-tiered boxes, in which they watched an item being hidden in one box and then asked to search for a similar item in the corresponding location at the second box. Half of the children heard a set of systematic terms conveying monotonic structure (1 2 3), whereas the other half of the children heard non-systematic terms consisting of names of familiar animals. Both sets of terms are familiar to preschoolers and neither directly denotes spatial locations. We found that preschool children who heard the 123 labels performed better than those who heard animal names. The results are evidence of young children’s sensitivity to the relational structure conveyed by language, and to their ability to apply this structure into a different domain.

Introduction
Relational similarity is a crucial construct in human cognition. The ability to perceive relational similarity underlies a number of fundamental cognitive processes such as analogies (Gentner, 1983, 2003), categorization (Ramscar and Pain, 1996), and inductive inferencing (Holland, Holyoak, Nisbett, and Thagard, 1986). How do children come to develop relational thinking? This question becomes especially relevant in the domain of spatial cognition – where an appreciation of spatial relations is necessary in a broad range of spatial tasks, from navigating to perceiving the configuration of a landmark array to map reading (e.g. Newcombe, 2002; Newcombe and Huttenlocher, 2000). Previous research has shown that the apprehension of relational similarities is not immediate in development: children first rely on overall similarity or on commonalities based on element matching, and then shift to appreciating commonalities based on relations (Gentner & Rattermann, 1991; Rattermann & Gentner, 1998a; Halford, 1993). This relational shift (Gentner, 1988) is also observed in the development of spatial cognition. For example, children understand element-to-element correspondences before they understand spatial relational correspondences (Bluestein & Acredolo, 1979; Presson, 1982). Liben (1998) also described a relational shift pattern in map understanding: children understood object-based correspondence before they understood relation-based correspondences. Blades and Cooke (1994) adapted the classic DeLoache task in which children watched a toy being hidden at an object in a model room and were asked to retrieve a similar toy hidden at the same object in a second model room. All objects (toy locations) were distinct except for one pair of identical objects. 3-year-olds succeeded when the toy was hidden at a unique object, but they failed the task when the toy was placed at one of the identical objects, suggesting a reliance on object-matching.

Gentner (2003) has proposed that the learning and application of relational language is a route to learning domain relations. Indeed, there is evidence suggesting that relational language might foster attending to and encoding particular relations (Hermer-Vasquez, Spelke, & Katsnelson, 1999; Logan & Sadler, 1996; Regier & Carlson-Radvansky, 2001). For example, in the development of spatial cognition, Hermer-Vasquez, Moffet, & Munkholm (2001) found that children’s performance in a search task was correlated with their ability to use the spatial language relevant to the task.

Recently, Loewenstein and Gentner (2005) found evidence from a spatial mapping task for the claim that spatial relational language fosters the development of spatial relational knowledge. Because our study is based on their methods we describe it in some detail. In the more challenging version of their task, using the cross-mapping technique—in which object matches do not correspond to relational matches—they found a semantically specific effect. Children who heard the set of spatial terms top, middle, bottom, performed significantly better than children who heard the set of spatial terms on, in, under. Loewenstein and Gentner conjectured that this difference in performance is due to the advantage of systematicity, that is, a connected system of relations such as top, middle, bottom invites a better relational representation, and this in turn supports relational mapping. The set of terms top, middle, bottom form a connected structure: top > middle > bottom. Thus they form an interconnected system governed by the
higher-order relation of monotonicity in the vertical dimension. In contrast, on, in, and under each conveys a separate first-order relation between the figure and the ground (Herskovits, 1986). They serve as three separate spatial relations rather than an interconnected system (although on and under can be seen as connected in some contexts). The structure mapping theory predicts that connected systems of relations should be favored over independent relations in analogical processing (Gentner, 1983; Clement & Gentner, 1991). Consistent with this prediction, hearing the terms top, middle, bottom should provide a deeper structural representation; and thus supports the mapping of spatial relations better than the less systematic set on, in, under.

The possibility that systematic relational language invites a systematic relational representation offers an appealing learning mechanism that could contribute to the development of higher-order cognition. But before embracing this conclusion, we must consider an alternative explanation for Loewenstein and Gentner results. It is possible that the set of terms on, in, under is more ambiguous in denoting specific locations within the box than is the top, middle, bottom set. For example, children might sometimes have interpreted “on the box” as “on the shelf,” which would lead to confusion between the top and the interior shelf. In contrast, the terms top, middle, bottom denote specific locations with greater clarity, e.g. the term top distinctly refers to the uppermost location of the box. Thus, the argument can be made that the difference between these two terms results not from the advantage of systematic relations, but rather from the advantage of greater referential specificity.

The current research tests whether systematic relational language can indeed affect children’s perceiving and encoding of relational structure. We used Loewenstein and Gentner’s (2005) spatial mapping task, but crucially, we used two sets of non spatial terms instead. One set of terms (123) represents a systematic relation of monotonic structure whereas the other set of terms does not (names of animals: sheep’s room, dog’s room and pig’s room). As neither set of terms is spatial, there is no a priori difference in the clarity of reference. In addition, in order to avoid possible canonical correspondences between our study set up and everyday observations, the three locations of the box were labeled 1, 2, 3 from top to bottom respectively, differing from the usual numbering of floors in buildings. We predicted that children who heard the set of terms 123 would perform better than those who heard names of animals.

It should be noted that our prediction is not at all obvious. Because children are ordinarily familiar with names of animals, one might expect that they will remember them better than numbers. In addition, young children may not have fully grasped the structure of the set of terms 123. Thus, one might well predict better performance with the set of animal names.

In sum, our study expands the original hypothesis of Loewenstein and Gentner by using general terms that embody monotonic structure rather than specifically spatial terms. In doing so, we aimed to investigate (i) children’s sensitivity to structures in the semantics of their language, and (ii) whether they can abstract this structure and apply it to tasks in another domain—in this case, the domain of space.

**Experiment**

**Participants.** Sixty-seven children participated in the experiment. There were three age groups, averaging 3;6 years (n = 26, range: 3;5 to 3;9 years), 4;1 years (n = 24, range: 3;11 to 4;3 years), and 4;7 years (n = 13, range: 4;5 to 4;9 years). Within each age group, children were randomly assigned to the Animal condition or to the 123 condition.

**Materials**

**Box Task.** Two boxes, a white Hiding Box and a blue Finding Box, were placed about 2 ft apart on the floor. The dimensions of each box were as follows: 15 in. high x 12 in. wide x 7 in. deep. The top, middle and bottom of the box were labeled as sheep’s room, dog’s room, and pig’s room respectively for the Animal condition, and number 1 room, number 2 room, and number 3 room respectively for the 123 condition.

An identical set of three colored cards (aquarium, earth, pizza) was created for each box. The card was placed in a clear acrylic 5 in. x 7 in. picture frame. The cards were placed in the box such that matching pictures were in mismatched locations (Fig. 1). For example, a card with pizza picture might be placed in the middle position in the Hiding Box and in the bottom position in the Finding Box. One of the cards in each box had a yellow star attached to its back, making it the “winner” card. At all times, there was a card placed at the top, middle and bottom of each box, only one of which was the winner. Two additional plain colored cards—a grey card for the Hiding Box and a blue card for the Finding Box, were used for the orientation phase and for a catch trial.

**Number Task.** Small plastic fish and zebras were used for the counting task at the end.

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1 To test children’s number knowledge, we also administered a number task which probed children’s understanding of the successor function (Sarnecka, Cerutti, & Carey, 2005).
Procedure

Training. The child was first shown the cards at the Hiding Box. Three picture cards were placed at the top, middle or bottom of the box. The plain white card was placed in front of the box. The star was affixed to the back of the white card—making this card the “winner”. The experimenter turned over each of the cards and asked the child if there was anything on the card, using specific labels for each location. For example, in the Animal condition the experimenter said, “Let’s look at the one in the sheep’s room. Does it have anything on the back?” Likewise, in the 123 condition, the term number 1 room (for example) would be applied. For the card with the star, the experimenter explained to the child that the particular card was a winner card, because it had a star on the back. The whole procedure was repeated at the Finding Box.

The experimenter then explained the mapping task. The child was told that the winners were always in the same place in the two boxes. Then the child was given a practice trial. The experimenter placed the winner to the right of the Hiding box, saying “I’m putting this winner next to the box.” The experimenter then put the winner at the Finding box in the corresponding place, saying: “And this winner goes right here, in the very same place.” The child was then asked to find the winner at the Finding box, and to retrieve the original winner at the Hiding box. This practice trial was used to provide a clear demonstration of how to play the game. The procedure and instruction for the practice trials were identical for the two conditions.

Search trials. As the child watched, the experimenter placed the winner at the top, middle or bottom of the box, saying either “I’m putting the winner in the sheep’s/pig’s/dog’s room” (Animal condition) or “I’m putting the winner in number 1/number 2/number 3 room” (123 condition). The experimenter then asked the child to close her eyes while hiding the other winner at the Finding box. The child opened her eyes and was asked to search for the winner at the Finding box, while reminded by the experimenter that the winner at the Finding Box was at the very same place as the winner at the Hiding Box. The child was allowed to search only once, and the experimenter showed the correct location of the card if the child searched wrongly. The child was tested at each location twice (non-consecutively) for a total of 6 trials. Two orderings of placements were used. Between the fourth and fifth trials there was a catch trial in which the winners were placed next to the boxes, just as in the practice trial. The catch trial was intended to ensure that the child understood and paid attention to the task.

Retrieval trials. After the child found the winner at the Finding Box, she retrieved the winner from the Hiding Box (i.e., the one they had seen being placed). This is a standard procedure for mapping tasks, and is typically used to assess children’s memory for the initial location (e.g., DeLoache, 1987).

Pointing task. To assess the child’s memory for the label of the spatial locations, at the end of the mapping task the child was asked to point to a particular labeled location. Thus for the Animal condition the experimenter asked the child “Can you point to the dog’s room?” or, in the 123 condition “Can you point to number 2 room?” All three spatial locations were assessed.

Open-ended question. After completing the mapping task, the child was asked: “How did you know where to look? How did you know which card would be the winner?”

Number knowledge task. Number task was administered to a subset of the participants. The standard Give-N-task was used. Children were presented with a wooden shallow box containing 20 small plastic zebras. The experimenter then asked the child to give her a certain number of zebras: “Can you give me __ zebras?” This task was then followed by a counting task where seven plastic fish were arranged in a line in front of the child. The experimenter then said “Now can you please count the fish?”

Scoring. The main dependent measure was the proportion of correct responses across the six search trials. Children who searched incorrectly in the catch trials were eliminated from the final analysis. This was to ensure that errors were not due to lack of attention or non-understanding of the task per se. There were no effects of gender, nor of order of presentation, nor was there any change in search or retrieval performance across trials.

Memory score. The memory score was assessed from the Pointing Task result. A score 1 was given if the child correctly pointed to the location asked by the experimenter. Since there were three locations, 3 is a perfect memory score.

Number knowledge score. The understanding of the successor function, that for a number x, the next number in line is x + 1, is important in understanding the structure of the integers. This understanding could be relevant for comprehending the systematicity of the set of terms 123. To estimate successor understanding, we followed Sarnecka, Cerutti and Carey’s (2005) “Give a number” method. They labeled children who could correctly give five or more items on request as the Cardinal-Principle knowers (CP-knowers), and children who could only give 1, 2, 3, or 4 items were considered as Subset knowers. Sarnecka et al. have shown that while both subset knowers and CP knowers can count roughly to 10, only the CP knowers performed significantly above chance on tasks that probed understanding of the successor function. These include the direction task – understanding that forward in the count list means increasing set size, and the unit task – understanding that a move of one step in the count list corresponds to a change of one item in the set size. Because knowledge of the cardinality principle correlates with understanding of the
successor function, we assessed performance on the “Give a number” task for a subset of children after they had completed the spatial mapping task.

Results

Search trials. 4;1-year-olds in the 123 condition performed significantly better ($M = .69$, $SD = .30$) than children in the Animal condition ($M = .44$, $SD = .28$), $F(1, 22) = 4.48$, $p < .05$. The 4;1-year-old children in the 123 condition performed significantly above chance ($t(11) = 4.21$, $p < .01$), but the children in Animal condition did not ($t(11) = 1.43$, $p = .18$). For 4;7-year-olds, there was no significant difference in performance between the 123 group ($M = .75$, $SD = .36$) and the Animal group ($M = .65$, $SD = .31$), $F(1, 15) = .48$, $p = .50$. Both groups performed significantly above chance, minimum $t(7) = 2.84$, $p < .05$. This suggests that for older children, overt use of language is no longer necessary for representing spatial relations. The youngest age group, 3;6-year-olds, also showed no difference between the 123 group ($M = .28$, $SD = .28$) and the Animal group ($M = .37$, $SD = .21$), $F(1, 24) = .91$, $p = .33$. However, in contrast to the 4;7-year-olds, neither the 123 nor the Animal groups performed significantly above chance.

Figure 2. Performance in Search Trials

* $p < 0.05$ against chance (0.33)
* $p < 0.05$ for differences between the two conditions

Retrieval trials. We found a high performance overall in retrieval trials for all age groups. A 2 (conditions) x 3 (age groups) ANOVA did not reveal any effect of age group, $F(2, 61) = 2.31$, $p = .11$), nor condition ($F(1, 61) = 2.55$, $p = .12$). There was no significant interaction of age group and condition, $F(2, 61) = .24$, $p = .79$. All three age groups performed reliably above chance in both conditions.

Memory score. We looked at whether the different sets of labels affect children’s memory of the names of the three locations on the box. A 3 (Age) x 2 (conditions) ANOVA revealed main effect of age ($F(2, 61) = 4.09$, $p < .05$). However, there was no main effect of condition ($F(1, 61) = .25$, $p = .62$) suggesting that there was no preference for either 123 label or sheep/dog/pig label to be used as a better mnemonic for differentiating the different locations in the box. Next, we compared 3;6-year-olds’ and 4;1-year-olds’ search trial performance in the two conditions given that they have a perfect memory score. For 3;6-year-olds, the perfect memory scorers in the 123 condition ($n = 5$) did not perform significantly different than those in the Animal group ($n = 7$), $F(1, 10) = .09$, $p = .77$. However, for the 4;1-year-olds with a perfect memory score, the 123 group performed significantly better than chance ($n = 7$, $M = .83$, $SD = .28$), $t(6) = 4.54$, $p < .005$, whereas the Animal group ($n = 4$, $M = .38$, $SD = .44$) did not perform better than chance, $t(3) = .21$, $p = .85$. This suggests that for 4;1-year-olds, even when remembering the different locations of the box is equally easy by hearing either number 123 or names of animal, the children who heard the set of terms with structured relation still benefited in the mapping task.

Number Knowledge Score. Across all age groups, we found no differences in mapping performances between the CP knowers and the subset knowers ($F(1, 45) = .14$, $p = .71$). It is possible that this test was too advanced to be sensitive to the simple ordinal structure that we are tapping into in our labeling system. That is, perhaps understanding the ordering relation among 123 does not require a general understanding of the successor function. As a comparison, in the counting task 90% of the children could count correctly above three, and majority of the children (73%) could count correctly beyond 7.

Discussion

We tested the hypothesis that relational language can influence relational representation. Specifically, we asked whether hearing a set of terms that convey a systematic relational structure would invite a correspondingly deep representation, which in turn would support the mapping of spatial relations. The most novel aspect of this research is our finding that the use of nonspatial terms (number 1/2/3 room) can support spatial representation and mapping in children as young as 4;1. These findings show that young children can be sensitive to structural relationships conveyed in the language they hear, and that they are able to abstract this structure into different domains.

Our results with 4;1 year olds support the hypothesis that hearing a set of terms conveying a systematic relational structure invites a representation sufficiently well-structured that the child can maintain a relational mapping even in the face of competing object matches.

Our findings are consistent with Loewenstein and Gentner’s (2005) finding of higher performance on a difficult cross-mapping task with top, middle, bottom (which conveys monotonic increase) than with the less...
systematic set on, in, under. Our results are also consistent with the pattern found by Gentner and Rattermann (1991; Rattermann & Gentner, 1998b). They found effects of introducing relational language on 3-year-old children’s ability to carry out a relational mapping. In these studies, the relational pattern was a monotonic increase in size across a line of objects; the correct answer was based on matching relative size and position. As in the present studies, the mapping was made difficult by introducing a cross-mapping between the object matches and the relational correspondences. The results showed that children who heard language conveying a monotonic relational structure (either big–little–tiny or Daddy–Mommy–Baby) performed far better than those who did not. The current findings add to evidence for a facilitating effect of relational language on children’s appreciation of relational similarities. Furthermore, our findings demonstrate that once young children acquire a grasp of the relations in a domain (such as the ordering relation within (123)), they are able to carry out a relational mapping between domains. At the same time, further analysis of the youngest age group suggests that there is a considerable degree of domain specificity at the outset.

Cross-domain vs. within-domain mapping

We can compare our current results with Loewenstein and Gentner’s results, as the two tasks were identical except for the set of terms that were used. Both studies utilized systematic set of terms that conveys a monotonic order: top middle bottom in L&G, and 123 in the current study. However, top middle bottom has the advantage of referring directly to spatial locations within the box, while 123 does not.2 Remarkably, for the 4-year-olds (4;1 in the current study, and 4;2 in L&G) the proportion of correct searches given 123 is almost the same as that in the top middle bottom group (M_{123} = .69, M_{TMB} = .65). This is compelling evidence for the power of language as a representational tool for highlighting and mapping relations.

However, among the younger group, we see a clear advantage for the more directly spatial terms, as would be predicted by the general finding of domain-specificity in very early learning. In our youngest age group (3;6), the numerical system 123 did not lead to insight: performance was at chance. In contrast, in L&G, the similar age group (3;7) who received top, middle, bottom performed significantly above chance. This does not appear to be the result of differential memory, as the children performed well in the retrieval task in both experiments. Rather, it suggests a developmental change in the ability to structurally align two different (but analogous) relational structures. The terms top middle bottom in L&G, which have direct spatial reference, could readily be mapped from box to box. But for the terms 1, 2, 3, the child had to apply and map a nonspatial to a spatial situation. It appears that the 3;6 group has not yet acquired the ability to perform such an abstract mapping.

Thus, the youngest children were able to benefit from within-domain, but not cross-domain relational language. This finding is consistent with the many prior studies suggesting that children’s learning begins with highly concrete, domain-specific representations (e.g., Mix, Sandhofer & Baroody, 2005). Why might this be? One possibility is that children lack sufficient processing capacity to carry out an abstract relational alignment (Halford, 1993). We think it more likely that their understanding of numerical structure is not yet firm enough to support such an abstract and difficult application. Although we did not see a difference in mapping performance between children based on whether they were able to pass the “give a number” test, it may be that a firm, generalized understanding of the successor principle requires more experience. It could also be the case that the “give-N” number task is insensitive to the kind of early gradation in number understanding that may apply here.

Developmental pattern

Another pattern that emerged from our results is that the effect of relational language disappears among the oldest group. While there was a clear advantage for 4;1-year-olds who heard the systematic terms, we found no such advantage for our oldest group (4;7 years). At this age, children in both groups performed equally well, and far above chance. This finding is consistent with Loewenstein and Gentner’s prediction that the benefits of overt relational language would disappear with age and experience. They conjectured that this would occur if over time the linguistic categories become habitual, so that the support of external language was no longer needed. Of course, a developmental change in reliance on language could also come about through the child developing nonlinguistic cognitive representations that support spatial reasoning.

While our developmental pattern suggests an attenuation of the effect of relational language in the older children, we are not claiming that the language effect is restricted to the initial period of acquisition. Obviously, the advantage of using overt relational language also depends on the difficulty of the task: for a more difficult task, even adults may still gain advantage from using language. For example, in a study by Wolff, Vassilieva, and Burgess (2002), people were given a mental rotation task involving spatial scenes. The results showed the typical pattern found in mental rotation studies (Shepard & Cooper, 1982): that is, reaction time increased with the degree of rotation through 180 degrees. However, when the spatial relations could readily be labeled, this typical pattern was no longer observed. Instead, people showed a fast, flat reaction time pattern, suggesting that the task was solved by using matching relational descriptions rather than via mental rotations.

Summary

The current results extend our understanding of how relational language may contribute to the development of relational cognition. Our results indicate that perceiving and mapping spatial relations can be enhanced not only by spatial relational language per se, but also by nonspatial

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2 Indeed, to avoid the child mapping from the floors of a building, we numbered the rooms in the reverse order from building floors; 1 for the top location, 2 for middle, 3 for bottom.
language that conveys a systematic structure of relations. It appears that, given linguistic support, young children can map relational structure from one domain to another. This offers a potential path by which well-structured knowledge can help to organize other domains in the course of development. We suggest that relational language plays an instrumental role in this transmission.

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